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Optimization of the imported air express cargo distribution problem

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Abstract

This study examines the delivering network of imported air express cargo as an integrated multi-depot vehicle routing problem. Integrated multi-depot vehicle routing problem attempts to decide which service centers should be used and how much freight should be unloaded in each service center. The role of an exchange point which is allowing the delivery vans and shuttles to exchange imported and exported goods is also addressed. Test results demonstrate the feasibility of the four models so these are highly promising for use in a diverse array of applications, such as in home delivery and reverse logistics.

Keywords: Air express cargo, Hub-and-spoke, Multi-depot vehicle routing, Exchange point

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1. Introduction

The air cargo industry typically comprises time-definite air express carriers and time-indefinite general airlines. Time-definite service offers premium door-to-door freight delivery for global shipping companies that expedite commerce by integrating the functions of pickup, customs clearance, air transportation and delivery (Lin et al., 2003). Air express cargo delivery closely resembles to airline scheduling, where packages are shipped daily from an origin to a destination. Under such circumstances, timing constraints are strictly controlled, because the service quality is normally guaranteed. According to Ho et al. (2008), product distribution from depots to customers is a challenging task in logistics management. Improved routing and scheduling decisions can increase customer satisfaction because additional customers can be served more efficiently. Moreover, product delivery at a reasonable cost is of priority concern among emerging e-businesses (Lee and Whang, 2001). The distribution problem is generally formulated as the vehicle routing problem (VRP). Effective distribution management encompasses a variety of decision making problems at strategic, tactical, and operational planning levels. On the operational level, various decisions concern the routing and scheduling of vehicles on a daily basis. The firm may route and schedule its vehicles to perform the assigned functions at a minimum cost.

This work presents a novel mathematical model that can determine the routes of both tractor-trailers and the routes of vans on a daily basis. Additionally, urban traffic congestion and the time windows of customers and airport are considered by applying the exchange points to the vehicle routing problem, referred to as the vehicle routing problem with exchange point (VRPE). An exchange point allows delivery/pick-up vans and shuttles to exchange cargo transported either from customers to a terminal or from a terminal to customers. In doing so, delivery vans may not have to return to service centers in order to load up imported goods. The firm may also not send the pick-up vans intentionally to collect exported cargo. This model attempts to minimize the overall transportation cost.

While viewing two parts of a delivering network as the related vehicle routing problem (VRP) and multi-depot vehicle routing problem (MDVRP), this study integrates them into a mathematical programming model, i.e., the integrated multi-depot vehicle routing problem (IMDVRP). IMDVRP attempts to resolve vehicle routing problems by determining which service center should be used and how much freight should unload in each service center. Firms can therefore adopt this model to schedule its vehicles. Additionally, the vehicle routing problem with exchange points can be extended to the multi-depot vehicle routing problem with exchange point (MDVRPE), which considers additional service centers. The MDVRPE is applicable to urban logistics given the severity of traffic congestion and scarcity of available parking. Finally, the IMDVRP model and MDVRPE model are combined as the integrated multi-depot vehicle routing problem with exchange point (IMDVRPE) to consider the fixed costs and transportation costs of various vehicles.

2. Problem description and pertinent literature

A transportation network of time-definite carriers comprises a service network and line-haul operations network. The service network includes pickup and delivery operations. Delivery involves carriers picking up shipments from shippers and then carriers delivering shipments to consignees. Both operations are performed by operating centers, each with a fleet of package cars. In between the two ends of delivery are the line-haul operations which move freight between pickup and delivery centers in a hub-and-spoke line-haul operations network.

Many distribution systems have adopted hub-and-spoke networks. According to Liu et al. (2003), in addition to fully exploiting the economies of scale in vehicle utilization, the hub-and-spoke system can also enhance customer service in terms of delivery frequency. Hubs function as transshipment points and facilitate the replacement of direct connections between all nodes with fewer, indirect connections. Reducing the total number of links decreases overhead costs not only by bundling flows, but also concentrating equipment and sorting at specific locations (Bryan and O'Kelly, 1999). The service network includes pickup of exported cargo and delivery of imported cargo. When imported cargo enters the air cargo terminal, and following sorting by service centers, the terminal send the shuttles with cargo to each area. The centers then send the vans from each center to customers. Exported cargo is collected from customers and service places to the airport by shuttles. Once daily air express shipments are received, each ground center unloads and reloads packages onto a fleet of ground vehicles for delivery. While packages are delivered to consignees, shippers pick up new shipments. While acting as a collection point for new shipments, the ground centers reload them onto ground line-haul tractor-trailers, i.e. ground feeders that transport the freight to air centers (Lin et al., 2003). Therefore, the operations network of tractor-trailers and the service network of vans can be viewed from the perspective of the vehicle routing problem (VRP).

Among the various forms that VRP can take based on the constraints and requirements of the network and the delivery demands include vehicle capacity, delivery time window, line-haul and back-haul demands, as well as multiple depots. Some studies have attempted to increase the efficiency of delivery systems by studying the design and operations of hub-and-spoke systems, in which the hub location is critical (Liu et al., 2003). Wasner and Zäpfel (2004) defined a depot as a consolidation center that bundles the quantities of parcels for certain demand points to achieve economies of scale for less-than-truckload (LTL) transport. As a consolidation center, a hub bundles quantities between depots to achieve economies of scale for depot-to-depot transports. This transportation system is characterized by an organizational structure in which single depots, i.e. terminals and spokes, encompass an area with specific collection and delivery points for each terminal. The depots are connected by at least one transshipment center or hub. The logistical flow is coordinated by bunching all shipments from one depot to another via a transshipment center (Zäpfel and Wasner, 2004).

The service network includes picking up exported cargo and delivering imported cargo. When imported cargo enters the air cargo terminal and the service centers sort it, the terminal sends the shuttles with cargo to each area. The centers then send the vans from each center to the customers. Shuttles collect exported cargo from customers and service places and then transport it to the airport. From the perspective of delivery, the hub of the international transportation network is assigned to ship and consign, while the center in the terminal sends the trailer-trailers with cargo to each service center; finally, vans ship the cargo to customers. Therefore, the operations network of tractor-trailers and the service network of vans can be handled as the Vehicle Routing Problem.

3. Mathematical formulations

3.1. Definitions and notations

IMDVRP attempts to schedule the vehicle route and determine which service centers are used and how much freight should be unloaded in each service center used. Meanwhile, VRPE focuses on determining whether to use the exchange points in delivery and, then if the points are used, determining how much freight should be delivered from the service center. Correspondingly, MDVRPE follows the same pattern. Figures 1-4 graphically represent these problems.

Notation and assumptions are summarized as follows:

1) *R* denotes the number of tractor-trailers and the index set of all tractor-trailers is {*1,..., r*}; *W* represents the number of shuttles and the index set of all Shuttles is {*r+1,..., W*}; and *V* refers to the number of vans and the index of all vans, as expressed by {*W+1,..., V*}.
2) M denotes the number of service centers, and {*1,..., M*} represents the index set of service centers; *n* refers to the number of customers, and {*M+1,...,n*} denotes the index set of customers: *N* represents the number of exchange points and {*n+1,..., N*} refers to the index set of exchange points; and *0* denotes the air cargo terminal.
3) Let X_{ij}^{k} be a binary variable and defined as follows: if arc $i-j$ is traversed by vehicle k, then $X_{ij}^{k}=1$; $X_{ij}^{k}=0$ otherwise. $k=\{1,2,\ldots,r\}$ is the tractor-trailer ; $k=\{r+1,\ldots,W\}$ is shuttle; $k=\{W+1,\ldots,W+V\}$ is van.
4) Let Z_{l}^{k} be a binary decision variable and defined as follows: if vehicle *k* travels from service center *l*, then $Z_{l}^{k}=1$; otherwise, $Z_{l}^{k}=0$.
5) Let e_{h}^{k} be a binary decision variable and defined as follows: the van *k*, if necessary, enters the exchange point *h*, then $e_{h}^{k}=1$; otherwise $e_{h}^{k}=0$.
6) Let y_{ij}^{k} be the amount of goods transported from node *i* to node *j* by van K. Additionally, $k=\{W+1,\ldots,W+V\}$.
7) The value T_{ij}^{k} is the carrying volume of Shuttles from node *i* to node *j*. Additionally, $k=\{r+1,\ldots,W\}$.
8) The value F_i denotes the delivery volume of each service centers used.
9) The value T_k denotes the carrying volume of van. Additionally, $k=W+1,\ldots,W+V$.
10) Let Q_k be the capacity of vehicle *k*. $k=\{1,2,\ldots,r\}$ are tractor-trailers ; $k=\{r+1,\ldots,W\}$ are shuttles; and $k=\{W+1,\ldots,W+V\}$ are vans.
11) Let d_i be the demand volume of customers.
12) C_{ij} denotes the cost incurred while the vehicle travels from *i* to *j*. In this study, we set $C_{ii}=0$.
13) Let S_i be the volume of each service center used should have.
14) This study considers the imported cargo, while excluding the exported cargo and time windows.
15) The delivered goods are the same products.

16) The distance is the Euclidean distance.

3.2. Mathematical Programming Formulations

3.2.1. Integrated multiple depots vehicle routing problem (IMDVRP)

The distribution of e-commerce problem as an integer program is expressed as follows:

The objective function of IMDVRP is as follows:

$$\sum_{k=1}^{r}\sum_{j=m+1}^{n} f_k X_{0j}^{k} + \sum_{i=1}^{m}\sum_{j=m+1}^{n}\sum_{k=r+1}^{V} f_k X_{ij}^{k} + \sum_{i=0}^{n}\sum_{i=0}^{n}\sum_{k=1}^{V} C_{ij} X_{ij}^{k} \qquad \text{(a-1)}$$

The above equation states the variable cost and fixed cost is to be minimized.

Arc constraints are as follows:

$$\sum_{k=1}^{r}\sum_{i=0}^{m} X_{ij}^{k} \le 1 \qquad j = 1,\dots,m \qquad \text{(a-2)}$$

$$\sum_{k=1}^{r}\sum_{j=0}^{m} X_{ij}^{k} \le 1 \qquad i = 1,\dots,m \qquad \text{(a-3)}$$

$$\sum_{j=1}^{m} X_{0j}^{k} \le 1 \qquad k = 1,\dots,r \qquad \text{(a-4)}$$

$$\sum_{i=1}^{m} X_{i0}^{k} \le 1 \qquad k = 1,\dots,r \qquad \text{(a-5)}$$

$$\sum_{k=r+1}^{r+V}\sum_{i=1}^{n} X_{ij}^{k} = 1 \qquad j = m+1,\dots,m+n \qquad \text{(a-6)}$$

$$\sum_{k=r+1}^{r+V}\sum_{j=1}^{n} X_{ij}^{k} = 1 \qquad i = m+1,\dots,m+n \qquad \text{(a-7)}$$

$$\sum_{i=1}^{m}\sum_{j=m+1}^{m+n} X_{ij}^{k} \le 1 \qquad k = r+1,\ldots,r+V \qquad \text{(a-8)}$$

$$\sum_{i=m+1}^{m+n}\sum_{j=1}^{m} X_{ij}^{k} \le 1 \qquad k = r+1,\ldots,r+V \qquad \text{(a-9)}$$

$$\sum_{i=0}^{m}\sum_{k=1}^{r} X_{ip}^{k} \le \sum_{j=m+1}^{n}\sum_{k=r+1}^{r+V} X_{pj}^{k} \qquad p = 1\ldots m \qquad \text{(a-10)}$$

$$\sum_{j=0}^{m} X_{0j}^{k} = 0 \qquad k = r+1,\ldots,r+V \qquad \text{(a-11)}$$

$$\sum_{i=1}^{m}\sum_{j=m+1}^{n} X_{ij}^{k} = 0 \qquad k = 1,\ldots,r \qquad \text{(a-12)}$$

$$X_{ii}^{k} = 0 \qquad \begin{array}{l} i = 0\ldots N \\ k = 1\ldots V \end{array} \qquad \text{(a-13)}$$

Equations (a-2) and (a-3) determine whether each center is served by one shuttle or not. Equations (a-4) and (a-5) guarantee that shuttle availability is not exceeded. Equations (a-6) and (a-7) suggest that each node is served by exactly one van. Equations (a-8) and (a-9) state that van availability is not exceeded. Equation (a-10) implies that service centers should be used when the tractor-trailers are passed by and vans go from that destination. Equation (a-11) states that the van cannot run the route of a shuttle. Restated, a van cannot run the route of an operation network. Equation (a-12) guarantees that the shuttle cannot run the route of a van. Restate, a shuttle cannot run the route of a service network. Equation (a-13) implies that no vehicle travels between the same nodes.

Route continuity constraints are as follows:

$$\sum_{\substack{i=0 \\ i\neq h}}^{m} X_{ih}^{k} - \sum_{\substack{j=0 \\ j\neq h}}^{m} X_{hi}^{k} = 0 \qquad \begin{array}{l} h = 0,\ldots,m \\ k = 1,\ldots,r \end{array} \qquad \text{(a-14)}$$

$$\sum_{\substack{i=1\\ i\neq q}}^{n} X_{iq}^{k} - \sum_{\substack{j=1\\ j\neq q}}^{n} X_{qi}^{k} = 0 \qquad \begin{array}{l} q=1,\dots,n \\ k=r+1,\dots,r+V \end{array} \qquad \text{(a-15)}$$

Equation (a-14) implies that, in order to achieve route continuity, a shuttle enters a node must exit from that same node. Equation (a-15) also implies that, in order to achieve route continuity, a van entering a node must exit from that same node.

Capacity constraints are as follows:

$$\sum_{k=r+1}^{r+V} \sum_{j=m+1}^{n} y_{ij}^{k} = F_i \qquad i=1,\dots,m \qquad \text{(a-16)}$$

$$\sum_{i=0}^{m}\sum_{k=1}^{r} y_{ip}^{k} - \sum_{j=0}^{m}\sum_{k=1}^{r} y_{pj}^{k} = F_p \qquad P=1,\dots,m \qquad \text{(a-17)}$$

$$\sum_{i=1}^{m} y_{i0}^{k} = 0 \qquad k=1,\dots,r \qquad \text{(a-18)}$$

$$\sum_{p=1}^{m} y_{0p}^{k} = \sum_{i=0}^{m}\sum_{p=1}^{m} X_{ip}^{k} \times F_p \qquad k=1,\dots,r \qquad \text{(a-19)}$$

$$y_{ij}^{k} \le x_{ij}^{k} \times Q_k \qquad \begin{array}{l} i=0,\dots,m \\ j=0,\dots,m \\ k=1,\dots,r \end{array} \qquad \text{(a-20)}$$

$$\sum_{i=1}^{n}\sum_{k=r+1}^{V} y_{ih}^{k} - \sum_{j=1}^{n}\sum_{k=r+1}^{V} y_{hj}^{k} = d_h \qquad h=m+1,\dots,m+n \qquad \text{(a-21)}$$

$$\sum_{i=m+1}^{m+n}\sum_{j=1}^{m} y_{ij}^{k} = 0 \qquad k=r+1,\dots,r+V \qquad \text{(a-22)}$$

$$\sum_{i=1}^{m}\sum_{h=m+1}^{m+n} y_{ih}^{k} = \sum_{i=1}^{n}\sum_{h=1}^{n} X_{ih}^{k} \times d_h \qquad k=r+1,\dots,r+V \qquad \text{(a-23)}$$

$$y_{ij}^{k} \le x_{ij}^{k} \times Q_{k} \qquad \begin{array}{l} i = m+1,\dots,n \\ j = m+1,\dots,n \\ k = r+1,\dots,V \end{array} \qquad \text{(a-24)}$$

Equation (a-16) specifies the volume that each service center should have. Equation (a-17) ensures that volume of each service center must be satisfied exactly. Equation (a-18) ensures that the shuttle must be an empty when arriving at the center. Equation (a-19) states that the freight from the air cargo terminal is the total freight of all service centers. Equation (a-20) specifies that the delivery volume cannot exceed the capacity of the shuttle. Equation (a-21) ensures that demand of each customer is satisfied precisely. Equation (a-22) ensures that the van is empty when returning to the service center. Equation (a-23) specifies that the volume from each service center is the total volume of all customer needs. Equation (a-24) states that the delivery volume cannot exceed the capacity of van.

Decision variables are as follows:

$$X_{ij}^{k} = 0 \; or \; 1 \qquad (for \; all \, i, j, k) \qquad \text{(a-25)}$$

$$y_{ij}^{k} \ge 0 \qquad (for \; all \, i, j, k) \qquad \text{(a-26)}$$

3.2.2. Vehicle Routing Problem with Exchange Points (VRPE)

The Vehicle Routing Problem with Exchange point is formulated as a mixed integer program, as described in the following. However, some notations differ from those listed above. Where 0 denotes the service center; *1~W* represents the Shuttle and *W+1~ W+V* refers to the van.

The objective function of VRPE is as follows:

$$\sum_{j=1}^{n}\sum_{k=W+1}^{V} f_k X_{0j}^{k} + \sum_{j=n+1}^{N}\sum_{k=1}^{W} f_k X_{0j}^{k} + \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=W+1}^{W+V} C_{ij} X_{ij}^{k} + 2 \times \left(\sum_{h=n+1}^{N}\sum_{k=1}^{W} C_{ij} X_{0h}^{k} \right) \qquad \text{(b-1)}$$

Arc constraints are as follows:

$$\sum_{k=W+1}^{W+V}\sum_{i=0}^{N} X_{ij}^{k} = 1 \qquad j = 1,\dots,n \qquad \text{(b-2)}$$

$$\sum_{h=n+1}^{n+N}\sum_{k=W+1}^{W+V} X_{hj}^{k} = 0 \qquad j = n+1,\dots,n+N \qquad \text{(b-8)}$$

$$\sum_{k=W+1}^{W+V}\sum_{j=0}^{N} X_{ij}^{k} = 1 \qquad i = 1,\dots,n \qquad \text{(b-3)}$$

$$\sum_{j=1}^{N} X_{0j}^{k} \le 1 \qquad k = W+1,\dots,W+V \qquad \text{(b-4)}$$

$$\sum_{i=1}^{N} X_{i0}^{k} \le 1 \qquad k = W+1,\dots,W+V \qquad \text{(b-5)}$$

$$\sum_{i=1}^{n}\sum_{h=n+1}^{n+N} X_{ij}^{k} \le 1 \qquad k = W+1,\dots,W+V \qquad \text{(b-6)}$$

$$\sum_{j=1}^{n}\sum_{h=n+1}^{n+N} X_{ij}^{k} \le 1 \qquad k = W+1,\dots,W+V \qquad \text{(b-7)}$$

$$X_{0h}^{k} = 0 \qquad \begin{matrix} h = n+1 \dots N \\ k = W+1 \dots V \end{matrix} \qquad \text{(b-9)}$$

$$X_{h0}^{k} = 0 \qquad \begin{matrix} h = n+1 \dots N \\ k = W+1 \dots V \end{matrix} \qquad \text{(b-10)}$$

$$X_{ll}^{k} = 0 \qquad \begin{matrix} l = 0 \dots N \\ k = 1 \dots V \end{matrix} \qquad \text{(b-11)}$$

$$\sum_{h=n+1}^{n+N} X_{oh}^{k} \le 1 \qquad k = 1,\dots,W \qquad \text{(b-12)}$$

$$\sum_{k=1}^{W} X_{0h}^{k} \times (V-W) \ge \sum_{i=1}^{n}\sum_{k=W+1}^{W+V} X_{ih}^{k} \qquad h = n+1,\dots,n+N \qquad \text{(b-13)}$$

Equations (b-2) and (b-3) ensure that each customer is served by exactly one van. Equations (b-4) and (b-5) guarantee that customer demand does not exceed van availability. Equation (b-6) and (b-7) state that each van has a specific time to enter the exchange point. Equation (b-8) ensures that vehicle cannot travel from one exchange point to another exchange point. Equation (b-9) ensures that van cannot travel directly from the service center to the exchange point. Equation (b-10) also ensures that a van cannot travel from an exchange point to a service center directly. Equation (b-11) implies that no vehicle travels between the same nodes. Equation (b-12) guarantees that customer demand does not exceed shuttle availability. Equation (b-13) ensures that when a van enters an exchange point, the shuttles must be in the exchange point.

Route continuity constraint is (b-14), while the sub tour breaking constraint is (b-15).

$$\sum_{\substack{i=0 \\ i\neq g}}^{N} X_{ig}^{k} - \sum_{\substack{j=0 \\ j\neq g}}^{N} X_{gj}^{k} = 0 \qquad \begin{matrix} g = 0,\dots,N \\ k = W+1,\dots,W+V \end{matrix} \qquad \text{(b-14)}$$

$$S = \left\{\left(X_{ij}^{k}\right) : P_i - P_j + N \times X_{ij}^{k} \le N-1\right\} \qquad \begin{matrix} 1 \le i \ne j \le N \\ k = W+1,\dots,W+V \end{matrix} \qquad \text{(b-15)}$$

Route continuity is represented by Equation (b-14), i.e. a vehicle that enters a demand node must exit from that same node.

Capacity constraints are as follows:

$$\sum_{i=0}^{N}\sum_{k=W+1}^{W+V} y_{iu}^{k} - \sum_{j=0}^{N}\sum_{k=W+1}^{W+V} y_{uj}^{k} = d_u \qquad u = 1,\ldots,n \qquad \text{(b-16)}$$

$$\sum_{j=1}^{N} y_{0j}^{k} = \sum_{i=1}^{n}\sum_{j=0}^{N} d_i \times X_{ij}^{k} \qquad k = W+1,\ldots,W+V \qquad \text{(b-17)}$$

$$y_{ij}^{k} \le X_{ij}^{k} \times Q_k \qquad \begin{array}{l} i = 0,\ldots,N \\ j = 1,\ldots,n \\ k = W+1,\ldots,W+V \end{array} \qquad \text{(b-18)}$$

$$\sum_{k=W+1}^{W+V} y_{0h}^{k} = \sum_{k=1}^{W} y_{0h}^{k} \qquad h = n+1,\ldots,n+N \qquad \text{(b-19)}$$

$$y_{0h}^{k} \le X_{0h}^{k} \times Q_k \qquad \begin{array}{l} h = n+1,\ldots,n+N \\ k = 1,2,\ldots,W \end{array} \qquad \text{(b-20)}$$

$$y_{ih}^{k} = 0 \qquad \begin{array}{l} k = W+1,\ldots,W+V \\ i = 1\ldots n \\ h = n+1\ldots N \end{array} \qquad \text{(b-21)}$$

$$y_{i0}^{k} = 0 \qquad \begin{array}{l} k = W+1,\ldots,W+V \\ i = 1\ldots n \end{array} \qquad \text{(b-22)}$$

$$\sum_{j=n+1}^{N} y_{ij}^{k} = 0 \qquad \begin{array}{l} i = n+1\ldots N \\ k = 1\ldots V \end{array} \qquad \text{(b-23)}$$

Equation (b-16) ensures each customer demand must be satisfied exactly. Equation (b-17) states that the volume from a depot is the volume of customer demands. Equation (b-18) states that arc flow cannot exceed the capacity of van. Equation (b-19) states that the total volume of vans carrying is the total volume of the shuttles carrying from service centers. Equation (b-20) states the volume which the shuttles carry cannot exceed its capacity. Equation (b-21) ensures a van must be empty arriving at an exchange point. Equation (b-

22) ensures a van must be empty when arriving at the service center. Equation (b-23) guarantees that no flow volume occurs between the exchange points.

Arc constraints and capacity constraints are as follows:

$$\sum_{j=1}^{N} X_{0j}^{k} = Z_0^k \qquad k = W+1 \ldots W+V \qquad \text{(b-24)}$$

$$y_{0j}^{k} \le Q_k \times Z_0^k \qquad j = 1 \ldots n, \quad k = W+1 \ldots W+V \qquad \text{(b-25)}$$

$$y_{0h}^{k} \le Q_k \times Z_0^k \qquad h = n+1 \ldots N, \quad k = W+1 \ldots W+V \qquad \text{(b-26)}$$

$$\sum_{i=1}^{n} X_{ih}^{k} = e_h^k \qquad h = n+1 \ldots N, \quad k = W+1 \ldots W+V \qquad \text{(b-27)}$$

$$y_{hj}^{k} \le Q_k \times e_h^k \qquad h = n+1 \ldots N, \quad j = 1 \ldots n, \quad k = W+1 \ldots W+V \qquad \text{(b-28)}$$

$$y_{0h}^{k} \le Q_k \times e_h^k \qquad h = n+1 \ldots N, \quad k = W+1 \ldots W+V \qquad \text{(b-29)}$$

Equation (b-24) implies that vehicle *k* travels from service center *l*. Equation (b-25) implies that the volume in which the van carries from the service center cannot exceed its capacity. Equation (b-26) guarantees that the volume in which the van carries from the service center to the exchange point cannot exceed its capacity. Equation (b-27) implies that van *k*, if necessary, enters exchange point *h*. Equation (b-28) suggests that the volume in which the van carries from the exchange point cannot exceed its capacity. Equation (b-29) guarantees that the volume in which the van carries from the service center to the exchange point cannot exceed its capacity.

Decision variables are as follows:

$$X_{ij}^{k} = 0 or 1 \qquad \textit{(for all i, j, k)} \qquad \text{(b-30)}$$

$$Z_0^k = 0 or 1 \qquad k = W+1 \ldots W+V \qquad \text{(b-31)}$$

$$e_h^k = 0 or 1 \qquad \begin{matrix} h = n+1 \ldots N \\ k = W+1 \ldots W+V \end{matrix} \qquad \text{(b-32)}$$

$$y_{ij}^k \geq 0 \qquad \textit{(for all i, j, k)} \qquad \text{(b-33)}$$

Let Z_0^k be a binary decision variable, and is defined as vehicle *k* travels from service center *0*, then $Z_0^k = 1$; otherwise, $Z_0^k = 0$. Let e_h^k be a binary decision variable, and is defined as van *k*, if necessary, enters the exchange point *h*, then $e_h^k = 1$; otherwise, $e_h^k = 0$.

3.2.3. Multiple Depots Vehicle Routing Problem with Exchange Points (MDVRPE)

The Objective function of MDVRPE is as follows:

$$\sum_{i=0}^{m} \sum_{j=m+1}^{n} \sum_{k=W+1}^{V} f_k X_{ij}^k + \sum_{i=0}^{m} \sum_{h=n+1}^{n+N} \sum_{k=1}^{W} f_k X_{ij}^k + \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=W+1}^{W+V} C_{ij} X_{ij}^k + 2 \times \left(\sum_{l=0}^{m} \sum_{h=n+1}^{n+N} \sum_{k=1}^{W} C_{ij} X_{lh}^k \right) \qquad \text{(c-1)}$$

The objective function (c-1) states that total variable cost and total fixed cost must be minimized.

Arc constraints, route continuity constraint, subtour breaking constraint, capacity constraints and decision variables are the same for the VRPH. Only two constraints are different.

$$\sum_{j=0}^{m} X_{lj}^k = 0 \qquad \begin{matrix} l = 0 \ldots m \\ k = 1 \ldots V \end{matrix} \qquad \text{(c-2)}$$

$$\sum_{j=0}^{m} y_{ij}^k = 0 \qquad \begin{matrix} i = 0 \ldots m \\ k = 1, \ldots, V \end{matrix} \qquad \text{(c-3)}$$

Equation (c-2) guarantees that no vehicle travels from the service center to another one. Equation (c-3) guarantees that no flow volume occurs between the exchange points.

3.2.4. Integrated Multiple Depots Vehicle Routing Problem with Exchange Points (IMDVRPE)

The objective function of IMDVRPE is as follows:

$$\sum_{k=1}^{r}\sum_{j=1}^{m} f_k X_{0j}^{k} + \sum_{i=1}^{m}\sum_{j=n+1}^{N}\sum_{k=r+1}^{W} f_k X_{ij}^{k} + \sum_{i=1}^{m}\sum_{j=m+1}^{n}\sum_{k=W+1}^{V} f_k X_{ij}^{k} + \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=1}^{V} C_{ij} X_{ij}^{k} + 2\times\left(\sum_{l=0}^{m}\sum_{h=n+1}^{n+N}\sum_{k=1}^{W} C_{ij} X_{lh}^{k}\right) \qquad \text{(d-1)}$$

The objective function (d-1) states that total variable cost and total fixed cost must be minimized. The constraints resemble those of IMDVRP and MDVRPH.

4. Experimental analysis

As is well known, routing and scheduling problems are NP-hard, implying that an efficient algorithm for solving the optimality problem has not been developed. Consequently, solving the problem via an exact algorithm is time consuming and computationally complex. By dealing with some of these small problems, this study demonstrates the effectiveness of the proposed methods. These problems are solved on a 600MHZ PC with Optimal Program Language (OPL) Studio, which uses the CPLEX-MIP Solver.

Example 1: IMDVRP

This example assumes that two tractor trailers and five vans are available, with each shuttle and each van having a capacity of 80 and 30 units, respectively. Table 1 lists the preliminary information of service depots and customer demands. The transportation cost is 523, while Table 2 lists the remaining results of this example.

Example 2: VRPE (1)

This example assumes that two shuttles and two vans are available, with each shuttle and each van having 50 and 30 units, respectively. In this example, the total capacity of all vans cannot satisfy all needs simultaneously and the time windows are not considered. Table 3 summarizes the preliminary information of service depots and customer needs. The minimum cost is 707. Table 4 summarizes the remaining results of this example.

Example 3: VRPE (2)

This case study assumes that two shuttles and four vans are available, with each shuttle and each van having a capacity of 50 and 30 units, respectively. In this case study, the total capacity of all vans can satisfy all demands at one time, implying that the VRPE model must decide whether to use the exchange point or not. The preliminary information of service centers and customers needs is the same as that with VRPE (1).The cost is 700. Table 5 summarizes the remaining results of this case study.

Example 4: VRP

Validity of the VRP is tested to compare VRPE (1) and (2). Whether or not the VRPE model can represent VRP must be determined. This case study assumes that four vans are available, with each van having the capacity of 30 units. The basic information of service centers and customer needs is the same as that with VRPE (1). The cost is 710. Table 6 lists the remaining results of this case study.

Example 5: MDVRPE (1)

This case study assumes that two service centers, three shuttles and three vans are available, with each shuttle and each van having the capacity of 50 and 30 units, respectively. In this case study, the total capacity of all vans cannot satisfy all needs at one time, implying that the VRPE model uses the exchange point. Table 7 lists the basic information of service centers and customer needs. Table 8 lists the remaining results of this case study.

Example 6: MDVRPE (2)

This case study assumes that two service centers, three shuttles and four vans are available, with each shuttle and each van having the capacity of 50 and 30 units, respectively. In this case study, the total capacity of all vans can satisfy all needs at one time, implying that VRPE model is allowed to decide whether or not to use the exchange point. The basic information of service depots and customer demands is the same as that with MDVRPE (1). The cost is 517. Table 9 lists the remaining results of this case study.

Example 7: MDVRP

Validity of the MDVRP is tested to compare the MDVRPE. Whether the MDVRPE model can represent MDVRP must be determined. This case study assumes that four vans are available, with each van having a capacity of 30 units. The basic information of service depots and customers demands is the same as that with MDVRPE (1). The cost is 530. Table 10 lists the remaining results of this case study.

Example 8: IMDVRPE (1)

This case study assumes that two tractor-trailers, two shuttles and five vans are available; each shuttle has the capacity of 80 units; each van has the capacity of 30 units. The cost is 523. Table 11 lists the results of this case study.

Example 9: IMDVRPE (2)

This example assumes that two tractor-trailers, two shuttles and two vans are available. Each Shuttle has the capacity of 80 units; each van has the capacity of 30 units. The basic information of service depots and customer demands is the same as that with MDVRP. The cost is 585. Table 12 summarizes the remaining results of this example. From the above experiments, we can infer that VRPE minimizes the traveling cost more than VRP does. Additionally, MDVRPE minimizes the traveling cost more than MDVRP does. Table 13 summarizes the comparison results.

5. Conclusions

Integrated global air express carriers offer time-guaranteed cargo delivery globally. The hub-and-spoke network and its variations consolidate partial loads, subsequently creating an efficient and extensively used network structure. This study formulates four mixed integer programming models of IMDVRP, VRPE, MDVRPE, and IMDVRPE. The IMDVRP model determines not only how to schedule a vehicle route, but also which service centers are used and how much fright should be unloaded at each service centers used. The VRPE and MDVRPE models determine the feasibility of using an exchange point to deliver and pick up imported and exported goods. The IMDVRPE model is an integrated model with IMDVRP and MDVRPE. The four models may be applicable to different areas. For instance, IMDVRP is promising for use in the automotive industry, while the VRPE, MDVRPE and IMDVRPE models may be applicable in urban logistics to resolve traffic congestion or other related problems. Based on those experimental results, we conclude that, while VRPE minimizes the traveling cost more than VRP does, MDVRPE minimizes the traveling cost more than MDVRP does.

This study also resolves problems involving a 600MHZ PC with OPL studio, which uses the CPLEX-MIP Solver. Test results demonstrate the feasibility of the four models. Despite its contributions, this study has certain limitations. While this study tested the validity of the four models based on small problems, a future study should design heuristic algorithms to solve them. Notably, the four models did not consider factors such as time windows, real world traffic situation and delay cost. Hopefully, a future study will address this issue. A model that incorporates more factors is more comprehensive. The four models developed in this study are highly promising for use in a diverse array of applications, such as in home delivery and reverse logistics.

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Table 1. IMDVRP: Locations and Demands

Node	X coordinate	Y coordinate	Demand	Node	X coordinate	Y coordinate	Demand
0.cargo terminal	0	0	-	6.customer	10	50	13
1.service center	0	40	-	7.customer	50	-40	6
2.service center	-40	0	-	8.customer	10	-50	14
3.service center	40	-20	-	9.customer	-30	-40	11
4.customer	-40	20	9	10.customer	30	30	5
5.customer	-50	20	10	11.customer	20	50	12

Table 2. IMDVRP: Delivering Amount and Vehicle Routing

Center	Amount	Vehicle	Carrying volume	Number	Routing
0. cargo terminal	80	1.Tractor-trailer (1)	80	2,1,3	0-2-1-3-0
		2.Tractor-trailer (2)	-	-	-
		Total	80		
		3.Van (1)	-	-	-
		4.Van (2)	-	-	-
2.service center	30	5.Van (3)	30	4,5,9	2-9-4-5-2
3.service center	20	6.Van (4)	20	7,8	3-7-8-3
1.service center	30	7.Van (5)	30	10,11,6	1-10-11-6-1
		Total	80		

Table 3. VRPE: Locations and Demands

Node	X coordinate	Y coordinate	Demand	Node	X coordinate	Y coordinate	Demand
1.service center	0	40	-	7.customer	50	-40	12
2.customer	-40	0	15	8.customer	10	-50	8
3.customer	40	-20	17	9.customer	-30	-40	20
4.customer	-40	-20	18	10.exchange point (1)	30	30	-
5.customer	-50	20	9	11.exchange point (2)	20	10	-
6.customer	10	50	10	12.exchange point (3)	0	-30	-

Table 4. VRPE (1): Vehicle Routings

vehicle	Carrying volume	Number	Routing
1.Shuttle (1)	50	12	1-12-1
2.Shuttle (2)	-	-	-
Total	50		
3.van (1)	53[29,24]	3,7,2,5	1-3-7-12-2-5-1
4.Van (2)	56[30,26]	6,9,8,4	1-6-9-12-8-4-1
Total	109		

Table 5. VRPE (2): Vehicle Routings

vehicle	Carrying volume	Number	Routing
1.Shuttle (1)	42	12	1-12-1
2.Shuttle (2)	-	-	-
Total	42		
3.Van (1)	10	6	1-6-1
4.Van (2)	-	-	
5.Van (3)	47[29,18]	3,7,4	1-3-7-12-4-1
6.Van (4)	52[28,24]	9,8,2,5	1-9-8-12-2-5-1
Total	109		

Table 6. VRP: Vehicle Routings

Vehicle	Carrying volume	Number	Routing
1.Van (1)	29	3,7	1-3-7-1
2.Van (2)	28	6,4	1-5-2-1
3.Van (3)	28	9,8	1-9-8-1
4.Van (4)	24	5,2	1-5-2-1
Total	109	-	-

Table 7. MDVRPE: Locations and Demands

Node	X coordinate	Demand	Y coordinate	Node	X coordinate	Y coordinate	Demand
1.service center	0	-	40	7.customer	50	-40	10
2.service center	-40	-	0	8.customer	10	-50	12
3.customer	40	15	-20	9.customer	-30	-40	8
4.customer	-40	17	20	10.customer	30	30	20
5.customer	-50	18	20	11.exchange point (1)	20	65	-
6.customer	10	9	50	12.exchange point (2)	-40	50	-

Table 8. MDVRPE (1): Delivering Amount and Vehicle Routing

Depot	Amount	vehicle	Carrying volume	Number	Routing
		1.Shuttle (1)	30	12	2-12-2
		2.Shuttle (2)	-	-	-
		3.Shuttle(3)	-	-	-
		Total	30		
1.service center	29	4.Van (1)	29	6,10	1-5-9-1
2.service center	80[25,25,30]	5.Van (2)	55[25,30]	3,7,8,5	2-3-7-12-8-5-2
		6.Van (3)	25	4,9	2-4-9-2

Table 9. MDVRPE (2): Delivering Amount and Vehicle Routings

Center	Amount	Vehicle	Carrying volume	Number	Routing
2.service center	80[18,17,45]	1. Shuttle (1)	25	12	2-12-2
		2. Shuttle (2)	-	-	-
		3. Shuttle (3)	-	-	-
		Total	25		
		4. Van (1)	18	5	2-5-2
		5. Van (2)	17	4	2-4-2
		6 van (3)	45[20,25]	8,9,7,3	2-9-8-12-6-3-2
1.service center	29	7 van (4)	29	6,10	1-6-10-1
		Total	109		

Table 10. MDVRP: Delivering Amount and Vehicle Routings

Depot	Amount	Vehicle	Carrying volume	Number	Routing
2.service center	55[25,30]	Van 4	30	5,8	2-5-8-2
		Van 5	25	4,9	2-4-9-2
1.service center	54[29,25]	Van 6	29	10,6	1-10-6-1
		Van 7	25	7,3	1-7-3-1
		Total	109		

Table 11. IMDVRPE (1): Delivering Amount Vehicle Routings

Depot	Amount	Vehicle	Carrying volume	Number	Routing
		1.Tractor-trailer (1)	-	-	-
0. cargo terminal	80	2.Tractor-trailer (2)	80	1,2,3	0-2-1-3-0
		Total	80		
		3.Shuttle (1)	-	-	-
		4.Shuttle (2)	-	-	-
		Total	-	-	-
		5.Van (3)	-	-	-
3.service center	20	6.Van (4)	20	8,7-	3-8-7-3
2.service center	30	7.Van (5)	30	4,5,9	2-4-5-9-2
1.service center	30	8.Van (6)	30	10,11,6	1-10-11-6-1
		9.Van (7)	-	-	-
		Total	80		

Table 12. IMDVRPE (2): Delivering Amount and Vehicle Routings

Depot	Amount	Vehicle	Carrying volume	Number	Routing
		1.Tractor-trailer (1)	-	-	-
0. cargo terminal	80	2.Tractor-trailer (2)	80	1,2	0-1-2-0
		Total	80		
		3. Shuttle (1)	-	-	-
		4. Shuttle (2)	30	12	2-12-2
		Total	-	-	-
1.service center	30	5. Van (1)	30	10,11,6	1-10-11-6-1
2.service center	50	6. Van (2)	50[20,30]	4,9,8,7	2-4-9-12-8-7-5-2
3.service center	-	Total	80		

Table 13. Comparison of different approaches

Example	Cost	Example	Cost	Example	Cost
IMDVRP	523	VRPE(1)	707	MDVRPE(1)	548
IMDVRPE(1)	523	VRPE(2)	700	MDVRPE(2)	517
IMDVRPE(2)	585	VRPE(3)	710	MDVRPE(3)	530
-	-	VRP	710	MDVRP	530

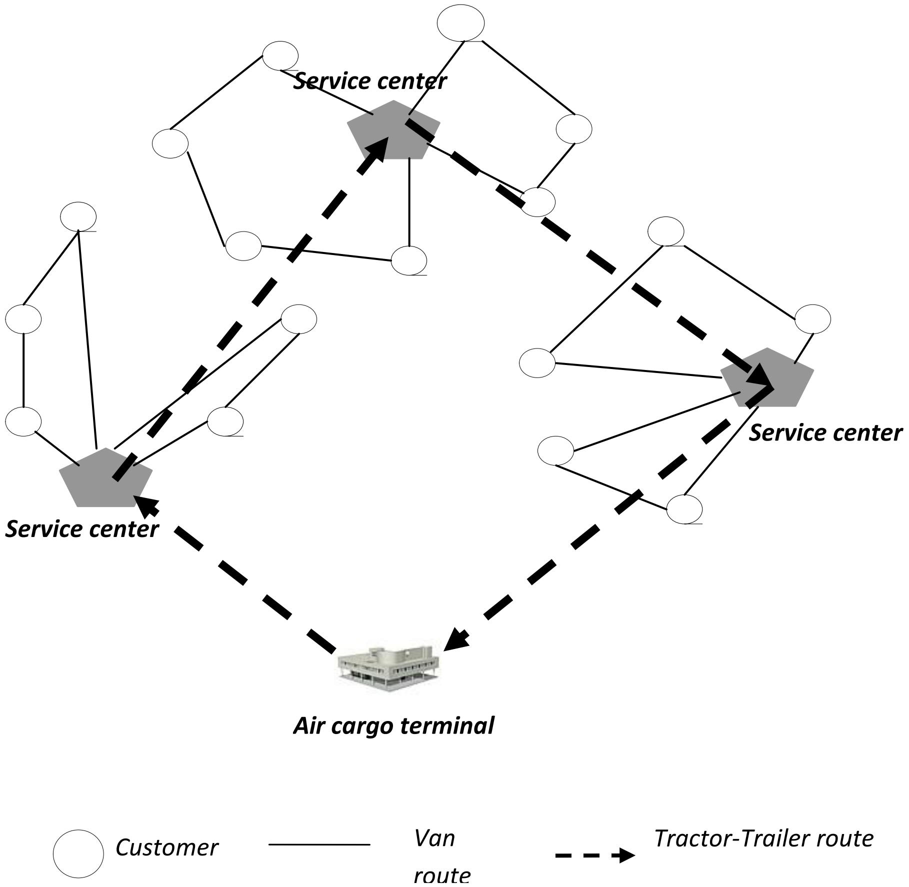

Figure 1. IMDVRP

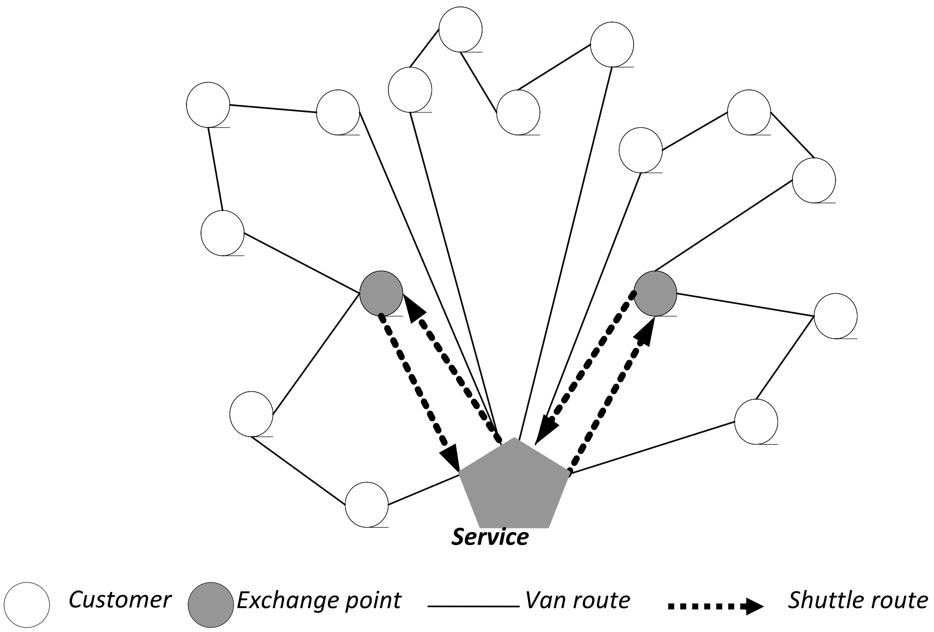

Figure 2. VRPE

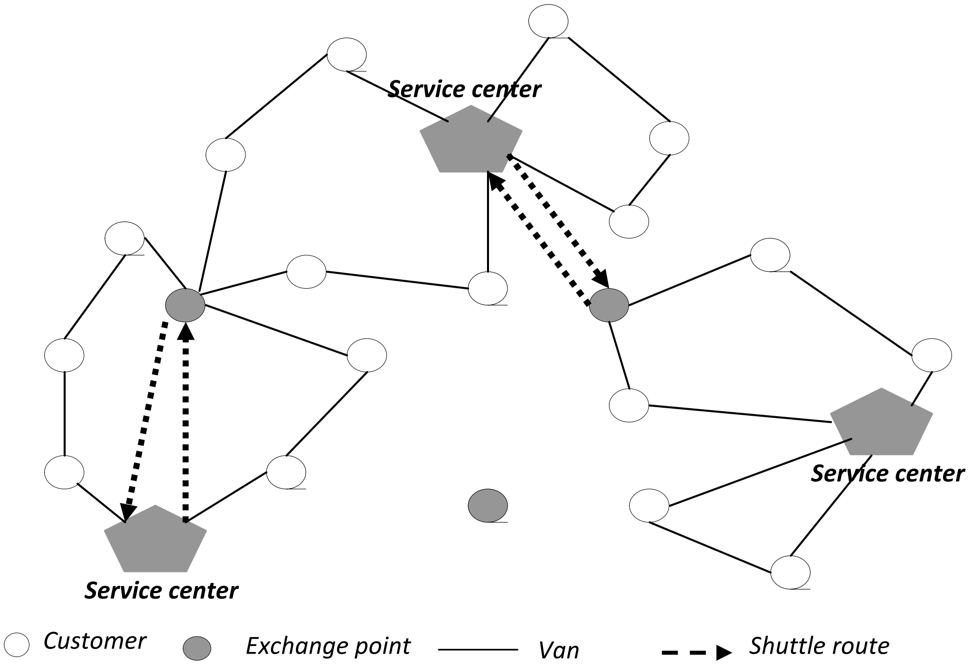

Figure 3. MDVRPE

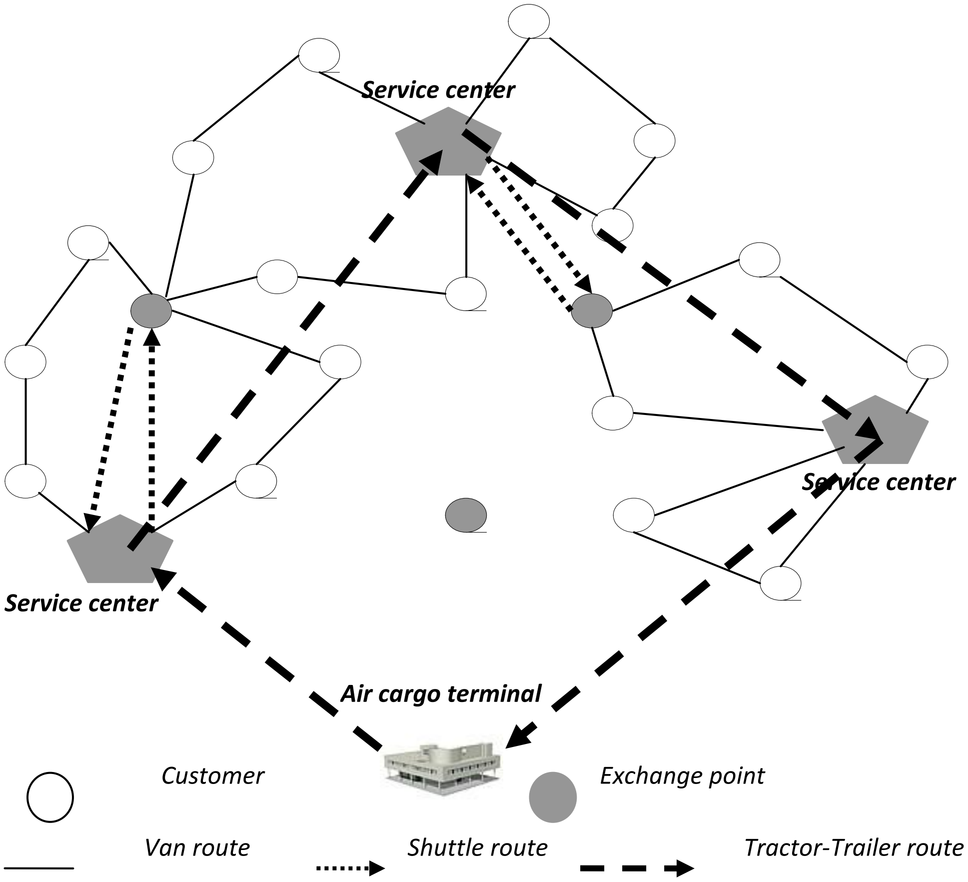

Figure 4. IMDVRPE